MAT1332 additional practice problems

The practice problems here are in addition to the material given in the suggested exercises, the assignments and the tests.
The problems here do NOT represent a sample exam.

1. Find the volume of the solid obtained by rotating the area bounded by $y = 4x - x^2$, $y = 3$, $x = 1$ and $x = 3$ about the x-axis.

2. Consider the system

$$\begin{aligned} x' &= \sqrt{3}x + 2y \\ y' &= 11x - \sqrt{3}y \end{aligned}$$

   (a) Show that $(0, 0)$ is the only equilibrium.
   (b) Write down the Jacobian matrix.
   (c) Show that the eigenvalues are $\lambda = \pm 5$.
   (d) For each eigenvalue, find the corresponding eigenvectors.

3. Evaluate $\int_1^4 e^{\sqrt{x}} dx$,

4. Evaluate $\int_1^2 \ln(x^2 e^x) dx$,

5. Evaluate $\int \frac{2x-1}{(x+4)(x+1)} dx$,

6. Evaluate $\int \frac{x^2+1}{x^2+3x+2} dx$,

7. Find the area enclosed by the following two functions $f(x) = \arctan x,\ g(x) = x$.

8. Does the integral $\int_0^2 \frac{1}{(1-x)^{1/3}} dx$ converge? If so, to which value?

9. Does the integral $\int_0^2 \frac{1}{(x-1)^4} dx$ converge? If so, to which value?

10. Does the integral $\int_0^2 \frac{1}{(x-1)^{2/5}} dx$ converge? If so, to which value?

11. Does the integral $\int_0^\infty \frac{1}{\sqrt{x+1}} dx$ converge? If so, to which value?

12. Solve $\frac{dx}{dt} = \sqrt{3t+1}$ with $x(0) = 1$.

13. Solve $\frac{dx}{dt} = 5 - 16t^2$ with $x(3) = -11$.

14. Solve $\frac{dy}{dx} = 3x^2 e^{2y}$ with $y(0) = 0$.

15. Solve $\frac{dy}{dx} = \frac{2x}{y+e^{5y}}$ with $y(2) = 0$.

16. Suppose that

$$\frac{dy}{dx} = y(1-y)(y-2)$$

(a) Find the equilibria of this differential equation.

(b) Graph $\frac{dy}{dx}$ as a function of y, and use your graph to discuss the stability of the equilibria.

(c) Draw the phase-line diagram.

(d) Use the derivative test to discuss the stability of equilibria.

(e) Sketch the solution with the initial conditions $y(0) = 0.5$ and $Y(0) = 1.5$ respectively.

17. Considering the following predator-prey system:

$$\begin{cases} \frac{dN}{dt} = aN(1 - \frac{N}{K}) - bNP \\ \frac{dP}{dt} = cNP - dP \end{cases}$$

where $N = N(t)$ is the prey density at time t and $P = P(t)$ is the predator density at time t. The constants a, b, c, d and K are positive. Furthermore, assume that $d/c < K$.

(a) Find all of equilibrium points.

(b) Find the Jacobian Matrix of the system in general.

(c) Evaluate the Jacobian Matrix at the nontrivial equilibrium (N^*, P^*) with $N^* > 0$ and $P^* > 0$, find its eigenvalues and determine whether this point is stable or not.

18. Find the tangent plane to the surface

$$z = f(x, y) = 4x^2 + y^2$$

at the point $(1, 2, 8)$.

19. Let

$$A = \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

(a) Show that the eigenvalues of A are -3 and -2.

(b) Find the solution of the system of differential equations

$$\frac{dx}{dt} = -4x(t) - 2x(t)$$
$$\frac{dy}{dt} = x(t) - y(t)$$

if $x(0) = 1$ and $y(0) = 2$.

(c) Draw the phase portrait of the system of differential equations given in (b).

20. Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 3 & -5 \\ 2 & 5 & -6 \end{bmatrix}$. Solve the system of linear equations $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.